

# Chiral tunneling through time-periodic potential in graphene

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## Abstract

Chiral tunneling through a harmonically driven potential barrier in graphene monolayer is considered in this work. Since the quasiparticles in this system are chiral in nature, tunneling is highly anisotropic, we determine the transmission probabilities for the central and sidebands as the incident angle of the electron beam is changed. Furthermore, we investigate how the transmission probabilities change as the width, amplitude and frequency of the oscillating barrier is changed. An interesting result of our study is that perfect transmission for normal incidence that has been reported for a static barrier persists for the oscillating barrier, manifestation of Klein tunneling in a time harmonic potential.

## I. INTRODUCTION

Advancement in technology has led to active investigation of electron transport in semiconductor nanostructures in time-dependent fields. The additional degree of freedom provided by the time dependence has led to the appearance of new phenomena in electron transport, for a review see [1] and references therein. Engineering of the confinement potential and band structure has allowed the possibility of studying photon assisted tunneling (PAT), where inelastic tunneling events occur in the presence of an ac field, in various driven systems. This topic is not only of academic interest but also has device applications. Early studies of PAT include the work of Dayem and Martin who provided evidence of absorption and emission of photons in tunneling transport in experiments on superconducting films in the presence of microwave fields [2]. Soon after this, Tien and Gordon theoretically justified this observation [3]. They assumed a time harmonic potential difference produced between the two films by a microwave field and solved the time-dependent Schrodinger equation for the system. Their photon assisted transport model accounted for transmission in the side bands in the presence of microwave radiation. The basic physical idea behind photon assisted tunneling is that an oscillating potential can lead to in-elastic tunneling where the electrons exchange energy quanta (photons) with the oscillating field. In such systems, a harmonically driven in time potential results in exchange of energy with electrons in the units of modulation quanta  $\hbar\omega$ ,  $\omega$  being the modulation frequency. Therefore, electrons at energy  $E$  can be transferred to the sidebands at energies  $E \pm n\hbar\omega$  ( $n = 0, \pm 1, \pm 2, \dots$ ) while traversing a region of space subjected to such a time-harmonic potential. The prototypical tunneling structure, which is an essential element of nanostructures where electron tunneling is investigated, is a single barrier. A common model in these studies is a time-modulated potential that has a finite spatial profile. Standard electron transport through various types of time-oscillating potential regions has been studied previously. More pertinent to the work undertaken here is that of Buttiker and Landauer. They investigated the traversal time of particles interacting with a barrier with time-oscillating height [4, 5]. Furthermore, M. Wagner wrote a series of papers on photon assisted transport through quantum wells and barriers with oscillating potentials [6]. Exchange of photons between the oscillating potential and electrons transfers electrons to the sidebands with a finite probability. Wagner determined these transmission probabilities using transfer matrix methods and discussed them as a function of the dimen-

sionless parameter  $\alpha$  which is the ratio of the amplitude of the time oscillating potential to its modulation energy. There are other contributions to this field that are relevant to our work and these have been put together in[7]. Recently, single layer carbon crystals (graphene monolayer) were fabricated which has generated considerable interest in finding a material that can replace silicon in micro-electronic devices. The idea of carbon based nano-electronics has been around since the discovery of carbon nanotubes. The recent fabrication of graphene monolayer has provided another avenue for carbon based electronics. Devices based on photon-assisted electron tunneling require the consideration of electron transport in time-harmonic potentials. For graphene based PAT devices it is essential to consider transport of charge carriers in graphene through time-harmonic potentials. To this end, we undertake the study presented here realizing that quasiparticles in graphene systems are quite different from the standard electrons that we encounter in conventional semiconductor based heterostructures. At low energies, quasiparticles (electrons and holes ) in graphene are described by the relativistic Dirac-like equation and possess charge conjugation symmetry as a single equation describes both particles (electrons) and antiparticles (holes). This is due to the crystal structure of graphene which is a layer of carbon atoms tightly packed in honeycomb lattice. It can be thought of as the superposition of two equivalent triangular sublattices conventionally called sublattice A and B. Quantum mechanical hopping between these sublattices results in the formation of two cosine-like energy bands. Intersection of these bands near the edges of Brillouin zone (Dirac points) leads to the conical energy spectrum  $E = \pm \hbar v_F k$  (with the effective Fermi speed (  $v_F = 10^6 m/s$ ). Above zero energy, the charge carriers in these systems are electrons which are usually termed Dirac electrons. The 2D Dirac-like spectrum was confirmed recently by cyclotron resonance measurements and also by angle resolved photoelectron spectroscopy (ARPES) measurements[8]. Recent theoretical work on graphene multilayers has also shown the existence of Dirac electrons with a linear energy spectrum in monolayer graphene[9]. The Dirac equation implies that the quasiparticles in graphene are chiral, tunneling through potential barriers in these systems is significantly different from systems where tunneling of standard electron occurs such as the two-dimensional electron gas (2DEG) systems realized in semiconductor heterostructures. Chiral nature of particles in graphene results in quantum tunneling being highly anisotropic where relativistic effects such as perfect transmission through high and wide barriers can occur (Klein tunneling[10])[11]. This occurs due to the conservation of chirality in inter-

action with the barrier, electrons in graphene can propagate to hole states through a high barrier without any damping. The study of this effect is relevant to the development of future graphene based devices. From a basic research point of view, graphene based systems, due to their lower ‘light speed’, can be quite useful for studying relativistic effects. Moreover, the role of chirality can be highlighted in electron transport in graphene. In graphene-based systems, electronic transport through barrier structures has been recently investigated [11, 12, 13, 14, 15, 16, 17, 18, 19]. In this work, we consider the transport of Dirac electrons in monolayer graphene through a harmonically oscillating in time square potential barrier. When standard electrons pass through a region which is subjected to time harmonic potentials, electronic transitions from central band to sidebands occur. Here, when transmission of Dirac electrons is considered, we also find transitions from the central to sidebands at energies  $E \pm n\hbar\omega$  ( $n = 0, \pm 1, \pm 2, \dots$ ) and determine the transmission probabilities for the sidebands. Moreover, we investigate how the transmission probabilities change as various parameters involved in the problem are varied with emphasis on the chiral nature of tunneling.

## II. FORMULATION

We consider monolayer graphene sheet in the  $xy$ -plane. The square potential barrier is taken to be in the  $x$ -direction while particles are free in the  $y$ -direction. Width of the barrier is  $a$ , height of the barrier is oscillating sinusoidally around  $V$  with amplitude  $V_1$  and frequency  $\omega$ . Electrons with energy  $E$  are incident from one side of the barrier in monolayer making an angle  $\phi_0$  with the  $x$ -axis and leave the barrier with energy  $E \pm n\hbar\omega$  ( $n = 0, \pm 1, \dots$ ) making angles  $\phi_n$  after transmission and  $\pi - \phi_n$  after reflection.

Let us consider the Hamiltonian  $H$  describing the system

$$H = H_0 + H_1 \tag{1}$$

where  $H_0$  is the Hamiltonian for the static case where the barrier height is not changing with time and  $H_1$  describes the harmonic time dependence of barrier height, given by

$$H_0 = -i\hbar v_F \sigma \cdot \nabla + V \tag{2}$$

$$H_1 = V_1 \cos(\omega t) \tag{3}$$

$V$ ,  $V_1$  are the static square potential barrier and the amplitude of the oscillating potential, respectively. Both  $V$  and  $V_1$  are constants for  $0 \leq x \leq a$  with  $a$  positive and are zero elsewhere.  $\sigma = (\sigma_x, \sigma_y)$  are the Pauli matrices,  $v_F$  is the Fermi velocity.

Solutions of the Dirac equation in the absence of the oscillating potential,  $H_0 \Psi = E\Psi$ , are given in [11] and can be used for constructing solutions to the time-dependent problem. For the tunneling problem, we consider the incoming electrons to be in plane wave states  $\Psi_i(x, y, t)$  at energy  $E$

$$\Psi_i(x, y, t) = e^{ik_y y} \begin{pmatrix} 1 \\ s_0 e^{i\phi_0} \end{pmatrix} e^{ik_1^0 x} e^{-iEt/\hbar} \quad (4)$$

where  $k_1^0$  and  $k_y$  are the  $x$ - and  $y$ -component of the electron wavevector, respectively.  $s_0 = \text{sgn}(E)$  and  $\phi_0$  is the angle that incident electrons make with the  $x$ -axis.

Reflected and transmitted waves have components at all energies  $E \pm l\hbar\omega$  ( $l = 0, \pm 1, \dots$ ) since the oscillating potential barrier can give and take energy away from electrons in units of  $\hbar\omega$ . This change in energy causes only the  $x$ -component of momentum to change. Hence, wavefunctions  $\Psi_r(x, y, t)$  for reflected and  $\Psi_t(x, y, t)$  for transmitted electrons, respectively are

$$\Psi_r(x, y, t) = e^{ik_y y} \sum_{l=-\infty}^{l=\infty} r_l \begin{pmatrix} 1 \\ -s_l e^{-i\phi_l} \end{pmatrix} e^{-ik_1^l x} e^{-i(E+l\hbar\omega)t/\hbar} \quad (5)$$

and

$$\Psi_t(x, y, t) = e^{ik_y y} \sum_{l=-\infty}^{l=\infty} t_l \begin{pmatrix} 1 \\ s_l e^{i\phi_l} \end{pmatrix} e^{ik_1^l x} e^{-i(E+l\hbar\omega)t/\hbar} \quad (6)$$

where

$$\begin{aligned} k_1^l &= \sqrt{\left(\frac{E + l\hbar\omega}{\hbar v_f}\right)^2 - k_y^2} \\ \phi_l &= \tan^{-1}(k_y/k_1^l) \\ s_l &= \text{sgn}(E + l\hbar\omega). \end{aligned}$$

In the barrier region, where  $H_1$  is nonzero, the eigenfunctions  $\Psi_b(x, y, t)$  of  $H$  can be expressed in terms of the eigenfunctions  $\Psi_0(x, y)$  of  $H_0$  as[3]

$$\Psi_b(x, y, t) = \Psi_0(x, y) \sum_{n=-\infty}^{n=\infty} J_n\left(\frac{V_1}{\hbar\omega}\right) e^{-in\omega t - iEt/\hbar}$$

where  $J_n\left(\frac{V_1}{\hbar\omega}\right)$  is the  $n$ th order Bessel function. A linear combination of wavefunctions at energies  $E + l\hbar\omega$  ( $l = 0, \pm 1, \dots$ ) has to be taken. Hence

$$\begin{aligned} \Psi_b(x, y, t) = e^{ik_y y} \sum_{l=-\infty}^{\infty} \left[ B_l \begin{pmatrix} 1 \\ s'_l e^{i\phi'_l} \end{pmatrix} e^{ik_2^l x} + C_l \begin{pmatrix} 1 \\ -s'_l e^{-i\phi'_l} \end{pmatrix} e^{-ik_2^l x} \right] \\ \times \sum_{n=-\infty}^{\infty} J_n \left( \frac{V_1}{\hbar\omega} \right) e^{-i(n+l)\omega t - iEt/\hbar} \end{aligned} \quad (7)$$

where

$$\begin{aligned} k_2^l &= \sqrt{\left( \frac{E - V + l\hbar\omega}{\hbar v_f} \right)^2 - k_y^2} \\ \phi'_l &= \tan^{-1}(k_y/k_2^l) \\ s'_l &= \text{Sgn}(E + l\hbar\omega - V). \end{aligned}$$

The wavefunctions given in equations(4-7) have to be continuous at the boundary. Applying this condition at  $x = 0$  and  $x = a$ , i.e.  $\Psi_i(0, y, t) + \Psi_r(0, y, t) = \Psi_b(0, y, t)$  and  $\Psi_t(a, y, t) = \Psi_b(a, y, t)$  and realizing that  $\{e^{in\omega t}\}$  are orthogonal, we obtain the following set of simultaneous equations:

$$A_n + r_n = \sum_{l=-\infty}^{\infty} [B_l + C_l] J_{n-l} \left( \frac{V_1}{\hbar\omega} \right) \quad (8)$$

$$A_n e^{i\phi_n} - r_n e^{-i\phi_n} = s_n \sum_{l=-\infty}^{\infty} \left[ B_l e^{i\phi'_l} - C_l e^{-i\phi'_l} \right] s'_l J_{n-l} \left( \frac{V_1}{\hbar\omega} \right) \quad (9)$$

here  $A_n = \delta_{n,0}$

$$t_n e^{ik_1^n a} = \sum_{l=-\infty}^{\infty} \left[ B_l e^{ik_2^l a} + C_l e^{-ik_2^l a} \right] J_{n-l} \left( \frac{V_1}{\hbar\omega} \right) \quad (10)$$

$$t_n e^{i\phi_n} e^{ik_1^n a} = s_n \sum_{l=-\infty}^{\infty} \left[ B_l e^{i\phi'_l} e^{ik_2^l a} - C_l e^{-i\phi'_l} e^{-ik_2^l a} \right] s'_l J_{n-l} \left( \frac{V_1}{\hbar\omega} \right). \quad (11)$$

The above set has infinite number of coupled equations and contains infinite number of unknowns(  $n, l$  goes from  $-\infty$  to  $\infty$  ). This linear system of equations cannot be analytically solved. Nevertheless, the infinite series in these coupled equations can be truncated and a finite number of terms starting from  $-N$  upto  $N$ , where  $N > \frac{V_1}{\hbar\omega}$ , retained if we note that the coupling strength is determined by the quantity  $\frac{V_1}{\hbar\omega}$  through Bessel functions  $J_n \left( \frac{V_1}{\hbar\omega} \right)$  and  $J_n \left( \frac{V_1}{\hbar\omega} \right)$ , they become negligible for order  $n$  higher than  $V_1 / \hbar\omega$ . Equations(8-11) are numerically solved for  $t_n$ . The transmission probability for the  $n$ th sideband,  $T_n$ , for which  $k_1^n$  is real and corresponds to propagating waves, is obtained from:

$$T_n = \frac{\cos \phi_n}{\cos \phi_0} |t_n|^2 \quad (12)$$

whereas imaginary  $k_1^n$  corresponds to evanescent waves that carry no particle current with the result  $T_n = 0$ .  $k_1^n$  can be real or imaginary depending on the particular values of the following parameters: incident energy  $E$ , oscillation frequency  $\omega$ , incident angle  $\phi_0$ . The numerical results obtained are discussed in the next section. Furthermore, analytical results are obtained if we consider small values of  $\alpha = \frac{V_1}{\hbar\omega}$  and include only the first two sidebands at energies  $E \pm \hbar\omega$  alongwith the central band at energy  $E$ . Moreover, we have to invoke the conditions  $\hbar\omega < E$  such that  $\text{sgn}(E \pm \hbar\omega) = +1$  and  $\hbar\omega < |E - V|$  such that  $\text{sgn}(E - V \pm \hbar\omega) = -1$  for  $E < V$ . Hence, we are able to truncate the sums in equations(8-11) retaining only the terms corresponding to the central and first sidebands and obtain analytical results for central and first sidebands,  $t_0$  and  $t_{\pm 1}$  :

$$t_0 = \frac{e^{-ik_1^0 a} \cos \theta_0 \cos \phi_0}{\cos \theta_0 \cos \phi_0 \cos[k_2^0 a] + i \sin[k_2^0 a](1 + \sin \theta_0 \sin \phi_0)}$$

$$t_n = \frac{1}{2} \frac{J_n(\alpha)}{J_0(\alpha)} \frac{t_{s0} t_{sn}}{\cos \phi_n} (\Gamma_n^+ + \Gamma_n^- e^{i(\phi_0 + \phi_n)} + \Delta_n (e^{i\phi_0} + e^{i\phi_n})) e^{i(\phi_n + k_1^0 a)}$$

where  $n = \pm 1$ ,  $t_{s0}$  and  $t_{sn}$  are transmission amplitudes for the static barrier at energy  $E$  and  $E + n\hbar\omega$  and

$$\Gamma_n^\pm = \Lambda_n^\pm - \Lambda_0^\pm,$$

$$\Lambda_n^\pm = \cos[k_2^n a \pm \theta_n] / \cos \theta_n,$$

$$\Delta_n = \Omega_n - \Omega_0,$$

$$\Omega_n = i \sin[k_2^n a] / \cos \theta_n.$$

In the high barrier limit,  $|V| \gg E$  with the result  $\theta_0, \theta_n \rightarrow 0$ , we obtain expressions for transmission probabilities for the central and the sidebands. For the central band

$$T_0 \approx \frac{\cos^2 \phi_0}{1 - \cos^2[k_2^0 a] \sin^2 \phi_0} = T_{s0} \quad (13)$$

where  $T_{s0}$  denotes the transmission probability at incident energy  $E$  and incident angle  $\phi_0$  in the case of the static barrier. This is the result obtained as Eq.(4) in[11]. For sidebands, we obtain:

$$\Lambda_n^\pm = \cos[k_2^n a] \Rightarrow \Gamma_n^\pm = -2 \sin[(k_2^n + k_2^0)a/2] \sin[(k_2^n - k_2^0)a/2]$$

$$\Omega_n = i \sin[k_2^n a] \Rightarrow \Delta_n = 2i \cos[(k_2^n + k_2^0)a/2] \sin[(k_2^n - k_2^0)a/2]$$

$$t_n = 2i \frac{J_n(\alpha)}{J_0(\alpha)} \frac{t_{s0} t_{sn}}{\cos \phi_n} \sin[(k_2^n - k_2^0)a/2] (\cos[(k_2^n + k_2^0)a/2] \cos[(\phi_n - \phi_0)/2] \\ + i \sin[(k_2^n + k_2^0)a/2] \cos[(\phi_n + \phi_0)/2]) e^{i(k_1^0 a + (\phi_0 - \phi_n)/2)}$$

The transmission probability for the sidebands is given by

$$T_n = \frac{\cos(\phi_n)}{\cos(\phi_0)} |t_n|^2 \\ = T_{s0} T_{sn} \left( 2 \frac{J_n(\alpha)}{J_0(\alpha)} \right)^2 \frac{\sin^2[(k_2^n - k_2^0)a/2]}{\cos \phi_n \cos \phi_0} (\cos^2[(k_2^n + k_2^0)a/2] \cos^2[(\phi_n - \phi_0)/2] \\ + \sin^2[(k_2^n + k_2^0)a/2] \cos^2[(\phi_n + \phi_0)/2])$$

where  $\hbar\omega < E \cos \phi_0$  otherwise  $T_{-1} = 0$ .  $T_{sn} = |t_{sn}|^2$  is the transmission probability of electrons at energy  $E + n\hbar\omega$  and incident angle  $\phi_n$  for the static barrier. We can also write the above expression as

$$T_n = T_{s0} T_{sn} \left( 2 \frac{J_n(\alpha)}{J_0(\alpha)} \right)^2 \frac{\sin^2[(k_2^n - k_2^0)a/2]}{\cos \phi_n \cos \phi_0} (\sin \phi_0 \sin \phi_1 \cos^2[(k_2^0 + k_2^1)a/2] + \cos^2[(\phi_0 + \phi_1)/2]). \quad (14)$$

At normal incidence,

$$T_{\pm 1} = \left( 2 \frac{J_{\pm 1}(\alpha)}{J_0(\alpha)} \right)^2 \sin^2[(k_2^0 - k_2^{\pm 1})a/2]$$

and if  $\hbar\omega < |E - V|$  we can write

$$k_2^0 - k_2^{\pm 1} = |E - V| / \hbar v_F - |E - V \pm \hbar\omega| / \hbar v_F = \pm \omega / v_F$$

with the result

$$T_{\pm 1} = \left( 2 \frac{J_{\pm 1}(\alpha)}{J_0(\alpha)} \right)^2 \sin^2 \left[ \frac{\omega a}{2v_F} \right] = \left( 2 \frac{J_{\pm 1}(\alpha)}{J_0(\alpha)} \right)^2 \sin^2[\omega\tau/2]$$

where  $\tau \equiv a/v_F$  is the time taken by a normally incident electron to cross the barrier *without multiple reflections* inside it. From the above expression, we note that  $T_1 = T_{-1}$ . For small  $\alpha$ ,  $J_{\pm 1}(\alpha) \approx \pm \alpha/2$ ;  $J_0(\alpha) \approx 1$  and  $\sin[\omega\tau/2] \approx \omega\tau/2$  when  $\omega\tau$  is small, corresponding to low frequency limit where frequency is smaller than the reciprocal of the traversal time. Using these results we obtain

$$T_{\pm 1} \approx \left( \frac{V_1}{2\hbar} \tau \right)^2$$



The above result can be compared with Eq.(8) in[4], where the transmission probability through a time-modulated barrier for the first sidebands is determined. The factor  $T$ , the transmission probability of the central band, is not unity and hence it appears there whereas  $T_{s0} = T_{sn} = 1$ , for normal incidence, in our case.

### A. Results and Discussions

The results for the transmission of Dirac electrons in graphene are now presented. The following parameters were used: The Fermi wavelength of the incident electron is taken to be  $\lambda = 50nm$ , the barrier oscillation frequency  $\omega = 5 \times 10^{12}Hz$ , the barrier width  $a = 100nm$  and the barrier height  $V = 200meV$ . The dependence of transmission probabilities on  $\alpha = V_1 / \hbar\omega$  for normally incident electrons and for those arriving at incident angle 30 degrees is shown in Figure(1a,b), respectively. For normal incidence, the angular dependence of the transmission probability for the  $n$ th sideband is independent of the sign of  $n$ :  $T_{+n} = T_{-n}$  for  $k_1^{-|n|}$  real. But this does not hold for incidence other than normal. We also find that the quantity  $\alpha$  is very significant in determining the relative transmission probabilities of various sidebands as shown in the figure. This implies that by adjusting the value of  $\alpha$  we can increase transmission through a particular sideband. It is seen that the central band dominates the transmission at all incident angles for small values of  $\alpha$  whereas contributions from higher and lower sidebands increases as  $\alpha$  becomes larger. This is plausible because for lower values of  $\alpha$  the oscillating barrier can be treated as a static one since we are keeping  $\omega$  fixed and changing  $V_1$  with the result that  $\alpha$  is proportional to  $V_1$  in these figures. Moreover, the total transmission probability through the central as well as the sidebands is unity. Hence, perfect transmission for the oscillating barrier at normal incidence which was earlier observed for the static barrier[11]. This is due to the chiral nature of the particles which results in perfect transmission (Klein tunneling).

In Figure(2a) we present the angular dependence of the transmission probability for the central-band  $T_0$  for various values of  $\alpha = V_1 / \hbar\omega$ . The transmission probability for the static barrier is also shown in the figure as it corresponds to  $\alpha = 0$ . The transmission probability  $T$  for the static barrier was previously obtained in[11]. We find resonant transmission through the oscillating barrier but unlike the static barrier we do not find perfect transmission for any incident angle. Realize that for the static barrier there is perfect transmission for certain

values of the incident angle. This is to be expected as the probabilities are now spread over the central and sidebands. In addition, the maximum transmission through the oscillating barrier depends on the value of  $\alpha$ .

Figure(2b) shows the transmission probabilities for the central band along with the first few sidebands as a function of the incident angle for  $\alpha = V_1 / \hbar\omega = 5$ . In this figure, we show how the incident particle flux is distributed in the sidebands (through the respective transmission probabilities) as the incident angle is varied. Note that the propagation angle for  $n$ th sideband is  $\phi_n$  which is not the same as the incident angle  $\phi_0$ . For this particular value of  $\alpha$ , transmission probability in the central band is very small for normal and close to normal incidence. For higher sidebands, more and more peaks in transmission probabilities occur. In the static case, the peaks in the transmission probability of the central band (there are no sidebands there) correspond to perfect transmission and the incident angles at which these occur can be obtained from the resonance condition,  $k_2^l = \frac{p\pi}{a}$  ( $p$  is an integer), through Eq.(13) and [11]. For the time-dependent situation being investigated here, it is not easy to determine the positions of the peaks as the analytic expression is more complicated. Nevertheless, we can understand how and where they occur by examining Eq (14), albeit for small  $\alpha$  where analytical results can be obtained but essential physics is the same. We observe, the transmission probability  $T_n$  given by Eq.(14) depends most strongly on the prefactor  $T_{s0}T_{sn}$  for the parameters considered here. The peaks correspond to the peak values of  $T_{s0}T_{sn}$ . Furthermore, the same behavior is seen for the static case as the transmission at higher incident energy there corresponds to transmission in the sidebands here. At these higher energies, the  $x$ -component of momentum in the barrier region satisfies the resonance condition greater number of times as the incident angle is varied, thus larger number of peaks.

We note that the absence of any potential gradient along the  $y$ -direction results in the conservation of the  $y$ -component of momentum. Therefore, change in energy that an electron experiences due to exchange of modulation quanta with the oscillating barrier brings about corresponding changes only in the  $x$ -component of the electron's momentum. For non-zero  $k_y$ , energy exchanges can make  $x$ -component of momentum imaginary inside or/and outside the barrier region that corresponds to unavailability of any energy state in the relevant region(s). If energy  $E + l\hbar\omega$  in the  $l$ th sideband is such that  $|E + l\hbar\omega| < \hbar v_F k_y$ , there are no propagating states available outside the barrier since  $k_1^l$  becomes imaginary. At

the same energy when particles have states available inside the barrier it can be localized if it is transferred to these states after losing energy through interaction with the oscillating barrier. In this situation, the particles are confined across the barrier while they are free to propagate along the barrier till one or more quantum of energy is absorbed, allowing transition to a higher sideband with states aligned in energy outside the barrier leading to eventual escape from the barrier region. For a graphene quantum well, confined electron states which arise due to the suppression of electron-hole conversion at the barrier have been discussed in [15].

For electron energy such that  $|E + l\hbar\omega - V| < \hbar v_F k_y$ , there are no propagating states available inside the barrier since  $k_2^l$  becomes imaginary. Furthermore, the energy at which electronic states outside the barrier match the hole states inside it, electronic transmission is governed by Klein tunneling while unavailability of hole states inside the barrier results in ordinary tunneling.

In Figure (3a) we present the transmission probability as a function of barrier width  $a$  for normal incidence. For the static barrier there is perfect transmission as can be seen in Figure (3a) where  $T$  represents the transmission probability for the static barrier whereas the transmission probability for the central band in the oscillating barrier decreases for smaller values of the barrier width and shows oscillatory but damped behavior for larger barrier width. The transmission probability for the other sidebands increases initially from zero but then oscillates with damped amplitude. We also observe that the contribution in transmission of the higher sidebands rises as the barrier width increases, this occurs due to larger time available to the electron for interacting with the oscillating potential as it traverses the barrier. In addition, we find that for normal incidence in the oscillating barrier:  $T_{+n} = T_{-n}$  for  $k_1^{-|n|}$  real. Nevertheless, the total transmission probability through the central as well as the sidebands is unity. These results imply that perfect transmission at normal incidence is independent of the barrier width, yet another manifestation of Klein tunneling.

In Figure(3b), the transmission probability as a function of barrier width  $a$  when the incident angle is 30 degrees is shown. The transmission probability represented by  $T$  for the static barrier now oscillates as a function of the barrier width whereas transmission probabilities for the central and sidebands in the oscillating barrier show behavior close to that obtained for normal incidence.

A comparison between analytical result obtained in Eq(14) and numerical results is presented in Figure(4) for  $\alpha = 0.5$ . Transmission probabilities  $T_{\pm 1}$  of first sidebands are plotted against incident angle  $\phi_0$ . Inset shows plot of  $T_{-1}$  versus  $\phi_0$ . It shows that transmission probabilities determined numerically exhibit the same behavior as obtained in the analytical result.

To summarize, we have considered the tunneling of chiral massless electrons corresponding to monolayer graphene through a barrier that is oscillating harmonically in time. We have determined how the transmission probability for the central and sidebands depends on the incident angle of the particles, the width of the barrier, the height and frequency with which it oscillates. Due to the chiral nature of the particles in graphene, tunneling is highly anisotropic with peculiar behavior at normal and close to normal incidence(Klein tunneling). We find, for normal incidence, perfect transmission in monolayer graphene. Klein tunneling that was observed for the static barrier is found to persist for the oscillating barrier.

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